

## ROLE OF DIFFERENTIAL EQUATIONS IN SOLVING ECONOMIC PROBLEMS

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### Abstract:

The article analyzed research in the field of models and equations used to analyze economic growth, investment, consumption and other important aspects of economics., as well as the role of differential equations in solving economic problems.

**Keywords:** Mathematics, optimization, differential, equation, economic, models.

### Introduction

Mathematics plays an important role in solving optimization models of economics. Optimization models help to find the optimal solution under constraint conditions, and mathematics provides tools and methods for formalizing and solving such models. Let's give an example of several ways in which mathematics is used in solving optimization models of economics, namely:

- Linear programming is one of the main optimization tools in economics. It is used to solve problems with linear constraints and a linear target function. Mathematical methods, such as the simplex method, are used to find the optimal solution.
- nonlinear programming is applied when the objective function or constraints are nonlinear. Mathematical methods such as gradient descent or least squares methods are used to solve such problems.
- Dynamic programming is used to optimize sequential solutions over time. Mathematical methods, such as Bellman's optimality principle, allow you to break a complex problem into simpler subtasks and find the optimal solution.

We conducted an analysis of some well-known works and authors who were engaged in research in the field of models and equations used to analyze economic growth, investment, consumption and other important aspects of the economy.

In "Economic Growth," Robert Solow introduced an economic growth model known as the Solow model, which became one of the major models in economic science. In another work, "Economic Growth and Capital Accumulation," Edward Denison conducted a study on the relationship between economic growth and capital accumulation using differential equations. Also, John Maynard Keynes in his work "The General Theory of Employment, Interest and Money" presented a consumption model that became the basis for analyzing economic growth and investment. In addition, Endogenic Technological Change Paul Romer

has developed an economic growth model that takes into account endogenous technological progress, which plays an important role in modeling economic growth.[1]

Robert Barro and Xavier Sala y Martin, in their study "A Contribution to the Theory of Economic Growth," introduced an economic growth model that takes into account human capital and its impact on economic growth.

In our paper, we will analyze the role of differential equations in solving economic problems. Differential equations are widely applied in various fields, such as physics, economics, biology, engineering, and others, to model and analyze dynamical systems and processes. They allow you to describe changes in quantities depending on their derivatives and interactions with other variables. Differential equations are mathematical equations that describe the dependencies between functions and their derivatives. They include both the function itself and its derivatives over one or more variables.

The general form of the differential equation is as follows:

$$F(x, y, y', y'', \dots, y^{(n)}) = 0,$$

where:

- x - independent variable,
- y - the desired function depending on x,
- y' is the first derivative of y over x,
- y'' - the second derivative of y in x,
- y<sup>(n)</sup> is the n-th derivative of y over x,
- F is the function that defines the relationship between y, its derivatives and x.

Differential equations can be classified according to various criteria, including the order of the equation (number of derivatives), the type of equation (ordinary, quotient, linear, nonlinear, etc.), and the conditions of the problem (initial or boundary conditions).

The solution to the differential equation is to find a function y (x) that satisfies the equation and possibly given initial or edge conditions. The solution can be explicit (analytic) or implicit (via parameterization or numerical methods).

Differential equations allow you to describe changes in quantities depending on their derivatives and interactions with other variables. Their solution is important for understanding and predicting the behavior of systems and processes in various fields of science and technology. Differential equations are a mathematical tool that describes the change of variables depending on their derivatives. In economics, differential equations play an important role in the modeling and analysis of various economic processes. They allow you to describe the dynamics of changes in economic variables over time and analyze their impact on economic phenomena.

For example, the Solow model can investigate the impact of changes in investment level, technological progress, and natural rate of population growth on economic growth. By changing the parameters in the Solow differential equation, it is possible to determine how the change in these factors will affect the rate of change in capital and therefore the level of production and economic growth. It is one of the most common differential equations in

growth models that describes the relationship between investment, production, and capital accumulation. The Solow equation has the form:

$$dK/dt = sY - (n + \delta)K,$$

where  $dK/dt$  is the rate of change in capital over time,  $s$  is the share of income that is invested in capital (savings ratio),  $Y$  is the level of production,  $n$  is the natural rate of population growth,  $\delta$  is the depreciation ratio (rate of decrease in capital over time).

Similarly, the Ramsay-Kass-Koopmans model can investigate the impact of investment, technological progress, and human capital on economic growth. By changing the parameters in the differential equations of this model, it is possible to determine how these factors affect the rate of change of capital and human capital, and therefore the level of production and economic growth. This is a more complex model that takes into account the impact of capital accumulation, technological progress and human capital on economic growth. This Ramsay-Kassa-Koopmans model takes into account the impact of capital accumulation, technological progress and human capital on economic growth.

The Ramsay-Kass-Koopmans equation can be of the form:  $dK/dt = sY - (n + \delta) K$ , where changes in human capital and technological progress are additionally taken into account.

In addition to the above, there is the Harrod-Domar model, which connects investment, capital and economic growth. The Harrod-Domar equation has the form:  $g = s/v$ , where  $g$  is the rate of economic growth,  $s$  is the share of income that is invested in capital,  $v$  is the capital coefficient.

As well as the Keynes consumption model, which describes consumption as a function of income. The Keynes consumption equation can be represented as  $C = c_0 + c_1Y$ , where  $C$  is consumption,  $Y$  is income,  $c_0$  and  $c_1$  are model parameters.

Thus, based on the above, differential equations allow you to simulate the dynamics of changes in economic variables over time. This is especially useful in studying the processes of growth, investment, consumption and other economic phenomena that change over time. Differential equations allow one to analyze how these variables change over time and what factors influence their dynamics.

The use of differential equations makes it possible to predict and plan economic phenomena and processes. For example, growth models based on differential equations make it possible to predict long-term trends in the economy and assess the impact of various factors on economic growth. This helps to make informed decisions and develop strategies to achieve the desired economic results.

Differential equations allow you to analyze the influence of various factors on economic variables. For example, using differential equations, you can estimate how a change in tax rate or investment policy can affect the level of production or consumption. This helps economists and decision-makers understand which factors are key to achieving certain economic goals.

Differential equations allow you to develop optimal strategies for solving economic problems. For example, differential equations can determine optimal investment strategies

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or policies to stimulate economic growth. This helps to optimize the use of resources and achieve the best results in the economy.

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