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PROBLEMS ABOUT THE LARGEST AND SMALLEST VALUES IN CIRCLES AND CIRCLES

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Abstract
This article provides solutions to some common problems related to circles and circles. Through these, the student learns that he can solve problems not in the same way, but also through different creative thinking.

Keywords: Circle, volume, proof, theorem of cosines, perimeter, bisector, angle, interior angle.

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Issue 1. Of all the triangles inscribed in a circle, find the smallest sum of the squares of the distances from the center of the circle to the sides of the triangle.

Solution: ABC – the desired triangle, a,b,c sides, distances from the center of the outer circle to the sides of the triangle, R - the radius of the outer circle.

$x = R\cos A, y = R\cos B, z = R\cos C$ we have

$x^2 + y^2 + z^2 = R^2(\cos^2 A + \cos^2 B + \cos^2 C) = R^2(3 - (\sin^2 A + \sin^2 B + \sin^2 C)) =$
$= 3R^2 - R^2(\sin^2 A + \sin^2 B + \sin^2(A+B))$ let's look at the total. Of course, $x^2 + y^2 + z^2$ collected, $\sin^2 A + \sin^2 B + \sin^2(A+B)$ is the smallest when the expression takes the largest value. We perform the following form substitutions:

$$\sin^2 A + \sin^2 B + \sin^2(A+B) = \frac{1-\cos 2A}{2} + \frac{1-\cos 2B}{2} + 1 - \cos^2(A+B) =$$
$$= \frac{4 - \cos 2A - \cos 2B - 2\cos^2(A+B)}{2} = 2 - (\cos(A+B)\times\cos(A-B) + \cos^2(A+B)) =$$
$$= 2 - (\cos(A+B) + \frac{1}{2}\cos(A-B))^2 + \frac{1}{4}\cos^2(A-B).$$

From this $\sin^2 A + \sin^2 B + \sin^2(A+B)$ for the expression to reach its greatest value, $\cos^2(A-B)$ it follows that it is necessary to take the largest value, i.e $\cos(A-B) = 1$ from that $(\cos(A+B) + \frac{1}{2}\cos(A-B))^2$ the expression must take the smallest value. So, we got the following system:

$$\begin{cases}\cos(A-B)=1\\ \cos(A+B)+\frac{1}{2}\cos(A-B)=0\end{cases} \text{ yoki } \begin{cases}\cos(A-B)=1\\ \cos(A+B)=-\frac{1}{2}.\end{cases}$$

From this $A-B=0, A+B=120^{\circ}$ yoki $A=B, A+B=120^{\circ}$. Finally, $A=B=60^{\circ}$ we find that So, the triangle you are looking for is a regular triangle.

Issue 2. A dot is given in a circle. Find the smallest distance passed through this point.

Solution: Given R we pass two wires through the point. These vatars AB and A_1B_1 through AB let it be perpendicular to the diameter of the shaft. A_1B_1 let it be voluntary. A_1B_1 home OR_1 we pass perpendicularly and OR_1R we make a right triangle. In this triangle OR gipotenuza OR_1 is greater than the cathet, so, AB watar A_1B_1 smaller than vatar. It follows that the circle is given R the smallest distance from a point is the distance from this point perpendicular to the diameter.

Issue 3. $2r$ A circle is inscribed in a triangle with constant perimeter. An attempt was made to parallel the side of the triangle to this circle. Find the smallest possible value of the cross-section between the sides of this triangle.

Solution: ABC – given triangle. This is the circle drawn inside the triangle according to the sides E, F, D try at the points. MN The attempt is common with the circle K has a point. $MK=ME, NF=NK$ will be. So, $MN=ME+NF$. Likewise $AC=AE+CF$. Based on these MBN the perimeter of the triangle

$2p-2AC$ is equal to. MBN And ABC because the triangles are similar

$\frac{MN}{AC}=\frac{p-AC}{P}$. $MN=y, AC=x$ we enter the designations. Based on these $\frac{y}{x}=\frac{p-x}{p}$. We solve this equation with respect to u and $y=\frac{1}{p}x(p-x)$ we will have. $\frac{1}{p}$ because it is a constant quantity, u variable $x(p-x)$ takes the largest value at the same time as multiplication. $x(p-x)$ and multiplication $x=(p-x)$, ya'ni $x=\frac{p}{2}$ takes the largest value when So, the attempted cross-section we are looking for takes the largest value in triangles whose base is equal to a quarter of the perimeter. There are an infinite number of such triangles. u we find the largest value of the variable $y_{max}=\frac{1}{p}\times\frac{p}{2}(p-\frac{p}{2})=\frac{p}{4}$.

From this, it follows that the intersection that is sought in the triangles in the problem condition is the middle line.

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Issue 4. In a given circle, a circle is drawn in such a way that the sum of the length of the circle and the length of the distance from the center of the circle to the circle is the largest.

Solution: Method 1. Looking for a home $AB = 2a$, O – distance from center to vatar $OD = d$ let it be

$c = 2a + d$ it is required to find the largest value of the expression. The radius of the circle R, $\angle BOD = \alpha$ let it be. BDO from a right triangle

$\begin{cases} a = R\sin\alpha \\ d = R\cos\alpha \end{cases}$ we find . The values of and in this system $c = 2a + d$ by putting,

$c = R(2\sin\alpha + \cos\alpha)$ we generate .

If 2 the $tg\varphi$ If we define by , then we have the following.

$c = \dfrac{R}{\cos\varphi}(\sin\alpha\sin\varphi + \cos\alpha\cos\varphi)$, or $c = \dfrac{R}{\cos\varphi}\cos(\alpha - \varphi)$. c variable

$\cos(\alpha - \varphi) = 1$ reaches its maximum when $\alpha - \varphi = 0 \Rightarrow \alpha = \varphi$ da. $tg\varphi = 2$ because

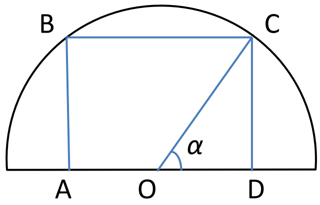
$\cos\alpha = \cos\varphi = \dfrac{1}{\sqrt{5}}$. And so, $c_{\max} = \dfrac{R}{\frac{1}{\sqrt{5}}} \cdot 1 = R\sqrt{5}$ is the length of the desired length

$2a = 2R\sin\alpha = 2R\sqrt{1 - \dfrac{1}{5}} = \dfrac{4R\sqrt{5}}{5}$ is equal to.

Method 2. Based on the designations in method 1 $d = \sqrt{R^2 - a^2}$, $d + 2a = c$ from $c = 2a + \sqrt{R^2 - a^2}$ yoki $5a^2 - 4ac + c^2 - R^2 = 0$ will be. This is when the discriminant of the last equation is zero s reaches a maximum, i.e $4c^2 - 5c^2 + 5R^2 = 0$ at From this $c_{\max} = R\sqrt{5}$. c using the value of a if we find $2a = \dfrac{4R\sqrt{5}}{5}$ will be equal to

Issue 5. A rectangle is drawn inside a semicircle. Two ends of this rectangle lie on the diameter, and the other two lie on the semicircle. A rectangle with the ratio of its sides will have the largest area.

Solving : OC – the radius of the semicircle , $\alpha - OC$ and the acute angle between the diameter. COD from a right triangle $CD = OC\sin\alpha$, $OD = OC\cos\alpha$ we find .



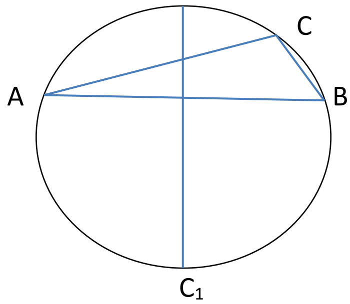
$ABCD$ rectangular face

$S = 2OD \times CD = 2OC\cos a \times OC\sin a = OC^2\sin 2a$ will be equal to S the largest value of the surface $\sin 2a = 1$ when, C_1

OC^2 will be equal to, i.e $a = 45^{\circ}$. So, $\dfrac{AD}{CD} = 2$.

Issue 6. Two in a circle A and B points are given. So C find the point such that the vatars $AC \times BC$ let the product be the largest.

Solution: If you are looking for C points AB if we take it on one side of the watar ,ABC angle takes the same values, this value a angle takes the same values, this value C_1 even if the situation of the point changes in any way (AB on one side from) $\dfrac{1}{2}AC \times BC \times \sin a$ expression ABC gives the face of the triangle. This is the surface $AC \times BC$ from magnitude, a constant multiplier $\dfrac{1}{2}\sin a$ differs from



ABC triangular face, $AC \times BC$ reaches its maximum value simultaneously with multiplication. But AB ohas an unchanging basis ABC the maximum value of the face of the triangle is at the maximum value of the height lowered to the base. From this C_1 point circle and AB it follows that it is the point of intersection of the middle perpendicular of the vatar. This mid-perpendicular intersects the circle at two points. From these points AB the one that is far away from will be the sought point. In our drawing it is C_1 there will be a point.

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