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NUMERICAL INVESTIGATION OF INTERACTION OF UNDERGROUND STRUCTURE (TUNNELS) WITH ELASTIC WAVES PROPAGATED IN GROUND MEDIUM

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ABSTRACT
In this article considers a dynamic method for calculating the analysis of an underground structure (tunnels) interacting with the soil environment. On the basis of numerical simulation, a diffraction pattern for an underground structure (tunnels) was constructed.

Keywords: finite element method, Newmark method, Bate method, stiffness matrix, damping matrix, mass matrix.

Introduction

Seismic waves - vibrations of rocks in the Earth, resulting from natural (earthquakes) or artificial processes of their excitation. At present, the study of seismic waves generated by earthquakes is necessary for understanding the nature of earthquakes and their prediction. Seismic waves caused by earthquakes or explosions elastic waves propagating in the body of the Earth whether. Of the body waves, the primary, or *P*, wave has the higher speed of propagation and so reaches a seismic recording station faster than the secondary, or *S*, wave. *P* waves, also called compressional or longitudinal waves, give the transmitting medium whether liquid, solid, or gas a back and forth motion in the direction of the path of propagation, thus stretching or compressing the medium as the wave passes any one point in a manner similar to that of sound waves in air.

Seismic action is a special term, which in the practice of calculating structures for seismic resistance means the oscillatory movement of the soil during an earthquake, which creates kinematic excitation of vibrations of building structures.

Literature Review

O. K. Zenkevich, The finite element method in engineering science. The monograph is devoted to the presentation of the fundamentals of the finite element method – one of the most effective modern methods for the numerical solution of engineering, physical and mathematical problems using computers.

L. Segerlind, Application of the finite element method. The monograph reflects the work of many researchers. The order in which the material is arranged depends on the results of the author's experience.

K. Bathe, Finite element procedures. Finite element procedures are now an important and frequently indispensable part of engineering analysis and design. Finite element computer programs are now widely used in practically all branches of engineering for the analysis of structures, solids, and fluids.

Formulation of the problem. The movement of an underground structure (tunnel) and its surrounding elastic medium (soil) is caused by the propagation of a longitudinal seismic wave in the medium in the area $\Omega = \{(x, y): x \in [0, L], y \in [0, H]\}$, and with external borders

$$\Gamma_l = \{(x, y): x = 0, y \in [0, H]\},$$
$$\Gamma_r = \{(x, y): x = L, y \in [0, H]\},$$
$$\Gamma_t = \{(x, y): x \in [0, L], y = H\},$$
$$\Gamma_b = \{(x, y): x \in [0, L], y = 0\},$$

as well as with the boundaries of an underground structure (tunnel) of a rectangular area

$$\Gamma_{lin} = \{(x, y): x = L1, y \in [0, H1 + H2]\},$$
$$\Gamma_{rin} = \{(x, y): x = [L1 + L2], y \in [0, H1 + H2]\},$$
$$\Gamma_{tin} = \{(x, y): x \in [0, L1 + L2], y = [H1 + H2]\},$$
$$\Gamma_{bin} = \{(x, y): x \in [0, L1 + L2], y = [0, H1]\},$$

The mathematical model in a strict formulation (differential equation) has the form:

$$\rho\ddot{u} + c\dot{u} - div(\sigma) = r \quad \text{in area } \Omega \tag{1}$$
$$\sigma_{xx} = c_p\rho\dot{u}, \sigma_{yy} = c_s\rho\dot{v}, \sigma_{xy} = 0 \text{ at the borders } \Gamma_t \text{ and } \Gamma_b \tag{2}$$
$$\sigma_{xx} = 2c_p\rho f(t)n_x, \sigma_{yy} = 2c_p\rho f(t)n_y, \sigma_{xy} = 0 \text{ at the border } \Gamma_l \tag{3}$$
$$\sigma_{xx} = c_p\rho\dot{u}, \sigma_{yy} = c_s\rho\dot{v}, \sigma_{xy} = 0 \text{ at the border } \Gamma_r \tag{4}$$
$$u = 0, \dot{u} = 0 \text{ при } t=o \tag{5}$$

Given the Koshi relation

$$\varepsilon_{xx} = \partial u_x/\partial x; \varepsilon_{yy} = \partial u_y/\partial y; \varepsilon_{xy} = \partial u_x/\partial y + \partial u_y/\partial x \tag{6}$$

and Hooke's law

$$\sigma = B\varepsilon,\ B = \begin{bmatrix} \lambda + 2\mu & \lambda & 0 \\ \lambda & \lambda + 2\mu & 0 \\ 0 & 0 & \mu \end{bmatrix} \tag{7}$$

λ and μ Lame coefficients.

In this problem, a longitudinal wave falls on the left edge at a given angle, with an initial impulse. Conditions for the transparency of borders have been established on all sides. Since the problem is solved by the finite element method, which involves setting a weak form (variational form) of the equilibrium equations, we will use the Lagrange variational principle, as well as the d'Alembert principle. As a result, the variational equilibrium equation for an isotropic body has the form:

$$\int_{\Omega} \lambda div(u) div(\delta u) + 2\mu\varepsilon^T\varepsilon - (r - \rho\ddot{u} - c\dot{u})\delta u d\Omega - \int_{\Gamma} P\,\delta u d\Gamma = 0 \quad (8)$$

where

r – volume load vector,

P – surface load vector.

Applying the method of partial discretization by finite elements i.e. movements in the element, representing in the form:

$$u^e = \sum_{i=1}^{n} u_i(t)\varphi(x,y)_i \quad (9)$$

where

n – the number of degrees of freedom in the element e,

$u_i(t)$ – desired nodal displacements depending on time,

$\varphi(x,y)_i$ – form functions on the element.

We obtain a system of linear ordinary equations:

$$M\ddot{U} + C\dot{U} + KU = R \quad (10)$$

With initial conditions

$$U = 0, \dot{U} = 0 \text{ at } t=0 \quad (11)$$

Equations (11) obtained from the consideration of static equilibrium at time t can be written as:

$$F_i(t) + F_d(t) + F_e(t) = R(t) \quad (12)$$

To solve systems of equations (12) with initial conditions (11), the implicit difference methods of Newmark and Bate are used.

Step-by-step solution using Newmark integration method.

A. Initial calculations:

1. Form stiffness matrix K, mass matrix M and damping constant C.
2. Initialize U_0, $\dot{U}_0$ and $\ddot{U}_0$.
3. Select time step Δt, and parameters α and δ and calculate integration constants:

$\delta \geq 0{,}50;\ \alpha \geq 0{,}25(0{,}5+\delta)^2;$

$a_0 = \frac{1}{\alpha\Delta t^2};\quad a_1 = \frac{\delta}{\alpha\Delta t};\quad a_2 = \frac{1}{\alpha\Delta t};\quad a_3 = \frac{1}{2\alpha} - 1;$

$a_4 = \frac{\delta}{\alpha} - 1;\quad a_5 = \frac{\Delta t}{2}(\frac{\delta}{\alpha} - 2);\quad a_6 = \Delta t(1-\delta);\quad a_7 = \delta\Delta t.$

4. Form effective stiffness matrix $\widehat{K}$:

$\widehat{K} = K + a_0 M + a_1 C.$

5. Triangularize $\widehat{K}$:

$\widehat{K} = LDL^t.$

B. For each time step:

1. Calculate effective loads at time $t+\Delta t$:

$\hat{R}_{t+\Delta t} = R_{t+\Delta t} + M(a_0 U_t + a_2 \dot{U}_t + a_3 \ddot{U}_t) + C(a_1 U_t + a_4 \dot{U}_t + a_5 \ddot{U}_t).$

2. Solve for displacements at time $t+\Delta t$:

$LDL^t U_{t+\Delta t} = \hat{R}_{t+\Delta t}.$

3. Calculate accelerations and velocities at time $t+\Delta t$:

$\ddot{U}_{t+\Delta t} = a_0(U_{t+\Delta t} - U_t) - a_2 \dot{U}_t - a_3 \ddot{U}_t;$

$\dot{U}_{t+\Delta t} = \dot{U}_t + a_6 \ddot{U}_t + a_7 \ddot{U}_{t+\Delta t}\ .$

Step-by-step solution using the Bathe integration method.

A. Initial calculations:

1. Form stiffness matrix K, mass matrix M and damping constant C.
2. Initialize U_0, $\dot{U}_0$ and $\ddot{U}_0$.
3. Select time step Δt and calculate integration constants:

$a_0 = \frac{16}{\Delta t^2};\quad a_1 = \frac{4}{\Delta t};\quad a_2 = \frac{9}{\Delta t^2};\quad a_3 = \frac{3}{\Delta t};$

$a_4 = 2a_1;\quad a_5 = \frac{12}{\Delta t^2};\quad a_6 = -\frac{3}{\Delta t^2};\quad a_7 = -\frac{1}{\Delta t}.$

4. Form effective stiffness matrices $\widehat{K_1}$ *and* $\widehat{K_2}$:

$\hat{K}_1 = K + a_0 M + a_1 C,$

$\hat{K}_2 = K + a_2 M + a_3 C.$

5. Triangularize $\widehat{K_1}$ *and* $\widehat{K_2}$:

$\hat{K}_1 = L_1 D_1 {L_1}^t,$

$\hat{K}_2 = L_2 D_2 {L_2}^t.$

B. For each time step:

First sub-step:

1. Calculate effective loads at time $t+\frac{\Delta t}{2}$:

$\hat{R}_{t+\frac{\Delta t}{2}} = R_{t+\frac{\Delta t}{2}} + M(a_0 U_t + a_4 \dot{U}_t + \ddot{U}_t) + C(a_1 U_t + \dot{U}_t).$

2. Solve for displacements at time $t+\frac{\Delta t}{2}$:

$L_1 D_1 {L_1}^t U_{t+\frac{\Delta t}{2}} = \hat{R}_{t+\frac{\Delta t}{2}}.$

3. Calculate accelerations and velocities at time $t+\frac{\Delta t}{2}$:

$\ddot{U}_{t+\frac{\Delta t}{2}} = a_1\left(\dot{U}_{t+\frac{\Delta t}{2}} - \dot{U}_t\right) - \ddot{U}_t;$

$\dot{U}_{t+\frac{\Delta t}{2}} = a_1\left(U_{t+\frac{\Delta t}{2}} - U_t\right) - \dot{U}_t\ .$

Second sub-step:

1. Calculate effective loads at time $t+\Delta t$:

$\hat{R}_{t+\Delta t} = R_{t+\Delta t} + M\left(a_5 U_{t+\frac{\Delta t}{2}} + a_6 U_t + a_1 \dot{U}_{t+\frac{\Delta t}{2}} + a_7 \dot{U}_t\right) + C(a_1 U_{t+\Delta t/2} + a_7 U_t).$

2. Solve for displacements at time $t+\Delta t$:

$L_2 D_2 {L_2}^t U_{t+\Delta t} = \hat{R}_{t+\Delta t}$.

3. Calculate accelerations and velocities at time $t + \Delta t$:

$\ddot{U}_{t+\Delta t} = -a_7 \dot{U}_t - a_1 \dot{U}_{t+\frac{\Delta t}{2}} + a_3 \dot{U}_{t+\Delta t}$,

$\dot{U}_{t+\Delta t} = -a_7 U_t - a_1 U_{t+\frac{\Delta t}{2}} + a_3 U_{t+\Delta t}$.

Analysis and results. The study was conducted on the following problem:
Meaning L=400m, H=100m, Young's modulus E=1.e8Pa, Poisson's ratio ν=0.3, $\eta = E/(2(1+\nu)),\ \lambda = \mathrm{E}\cdot\nu/((1+\nu)\cdot(1-2\cdot\nu))$ - Lame coefficients, density ρ=1800Pa, coefficient of the longitudinal and transverse waves of the medium were calculated by the formula $c_p = \sqrt{\mu/\rho},\ c_s = \sqrt{(\lambda+2\mu)/\rho}$.

A triangular element with six nodes, the second degree of accuracy, was used as finite elements.

A study was carried out on the influence of the number of elements on the accuracy of the solution, the following table shows the value of the displacements in the center of the two-dimensional region at the point (200,50), the calculation was carried out by the Newmark method, with a step dt=0.001sec at a point in time t=1sec:

Amount of elements\Bias	212(20/5)	858(40/10)	1896(60/15)	5306(100/25)
u_x	-0.00728348	-0.00739348	-0.00742446	-0.00743622
u_y	0.00130321	0.00126898	0.00129406	0.00129469

It can be seen from the table that with an increase in the partition, the accuracy of the result increases and when the sides are divided into 100/25 we have 4 correct digits.

Next, a comparative analysis of the calculations by the Newmark method and the Bathe method was carried out.

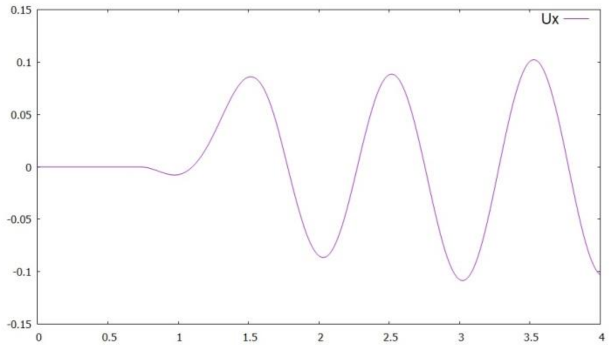
The following table shows the offset values u_x at the point (200,50) at a point in time t=1 sec. with the same physical parameters as above.

Step\ Method	0.05	0.01	0.005	0.001	0.0001
Newmark	-0.006348	-0.007408	-0.007421	-0.007436	-0.007424
Bathe	-0.006994	-0.007417	-0.007421	-0.007424	-0.007424

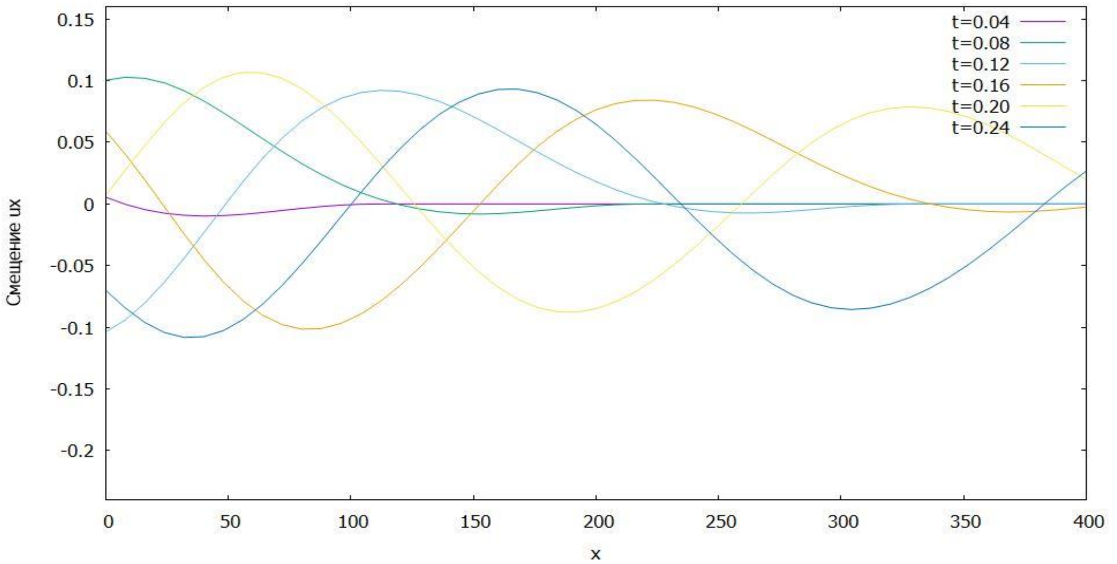
It can be seen from the table that at a step 0.0001 Newmark's method yields an exact solution, while Bathe's method yields an exact solution with a step 0.001. Thus, the Bathe method produces a more accurate result with a larger step.

Next, consider the oscillation of the midpoint of a two-dimensional body with the above physical and geometric characteristics.

The figure shows the movement of a point for 4 seconds.



The dynamics of wave propagation along the line y=50 is shown in the following figure



Conclusion

Studies have shown that for dynamic calculation it is not necessary to thicken the mesh, the use of the Bate method is preferable to the use of the Newmark method, the use of the boundary transparency condition correctly simulates the passage of the wave.

References

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